

Aufgabe 1

$$(i) \lambda_1 \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} + \lambda_3 \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$$\rightarrow \left( \begin{array}{ccc|c} 5 & 1 & 3 & 2 \\ 0 & -2 & -1 & 1 \\ 4 & -1 & 2 & 1 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 5 & 1 & 3 & 2 \\ 4 & -1 & 2 & 1 \\ 0 & -2 & -1 & 1 \end{array} \right) \begin{array}{l} \\ 5z_2 - 4z_1 \\ \end{array}$$

$$\rightarrow \left( \begin{array}{ccc|c} 5 & 1 & 3 & 2 \\ 0 & -2 & -1 & 1 \\ 0 & -2 & -1 & 1 \end{array} \right) \xrightarrow{z_3 - z_2} \left( \begin{array}{ccc|c} 5 & 1 & 3 & 2 \\ 0 & -2 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\rightarrow -5\lambda_3 = 0 \rightarrow \lambda_3 = 0$$

$$\rightarrow -3\lambda_2 - 2\lambda_3 = 1 \rightarrow -3\lambda_2 = 1 + 2\lambda_3 = 1 + 0 = 1$$

$$\rightarrow -3\lambda_2 = 1 \rightarrow \lambda_2 = -\frac{1}{3}$$

$$\rightarrow 5\lambda_1 + 1 - 3 = 2 \rightarrow 5\lambda_1 = 4 \rightarrow \lambda_1 = \frac{4}{5}$$

$$\underline{\vec{a} = 2\vec{b} + \vec{c} - 3\vec{d}}$$

$$(ii) \lambda_1 \begin{pmatrix} 1 \\ 2 \\ 0 \\ \vec{b} \end{pmatrix} + \lambda_2 \begin{pmatrix} 4 \\ 2 \\ 1 \\ \vec{c} \end{pmatrix} + \lambda_3 \begin{pmatrix} 2 \\ 5 \\ 1 \\ \vec{d} \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ -1 \\ \vec{a} \end{pmatrix}$$

$$\left( \begin{array}{ccc|c} 1 & 4 & 2 & -3 \\ 2 & 2 & 5 & 0 \\ 0 & 1 & 1 & -1 \end{array} \right) \xrightarrow{2z_1 - z_2} \left( \begin{array}{ccc|c} 1 & 4 & 2 & -3 \\ 0 & 6 & -1 & -6 \\ 0 & 1 & 1 & -1 \end{array} \right) \xrightarrow{6z_3 - z_2} \left( \begin{array}{ccc|c} 1 & 4 & 2 & -3 \\ 0 & 6 & -1 & -6 \\ 0 & 0 & 7 & 0 \end{array} \right)$$

$$\rightarrow -7\lambda_3 = 0 \rightarrow \lambda_3 = 0$$

$$\begin{array}{l} 6\lambda_2 = -6 \rightarrow \lambda_2 = -1 \\ \lambda_1 - 4 = -3 \rightarrow \lambda_1 = 1 \\ \rightarrow \vec{a} = 2\vec{b} - \vec{c} \end{array}$$