Aufgabe 6.

(i) $\vec{a} = 2\vec{e_x} - \vec{e_y} - 2\vec{e_z}$

$\vec{a} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

$|\vec{a}| = \sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{9} = 3$

$\vec{e_a} = \begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{2}{3} \end{pmatrix} = \frac{\vec{a}}{|\vec{a}|}$

$\cos(\varphi_x) = \frac{a_x}{|\vec{a}|} = \frac{2}{3}$

$\cos(\varphi_y) = \frac{a_y}{|\vec{a}|} = -\frac{1}{3}$

$\cos(\varphi_z) = \frac{a_z}{|\vec{a}|} = -\frac{2}{3}$

(ii) $\vec{a} = \begin{pmatrix} 4 \\ 2 \\ -5 \end{pmatrix}$; $\vec{b} = \begin{pmatrix} -2 \\ 7 \\ 4 \end{pmatrix}$

$\sphericalangle(\vec{a}; \vec{b}) = \alpha$

$\cos(\alpha) = \frac{\vec{a} \circ \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{4 \cdot (-2) + 2 \cdot 7 + (-5) \cdot 4}{\sqrt{4^2 + 2^2 + (-5)^2} \cdot \sqrt{(-2)^2 + 7^2 + 4^2}}$

$\cos(\alpha) = \frac{-14}{\sqrt{45} \cdot \sqrt{69}} = \frac{-14}{3\sqrt{345}}$

$\Rightarrow \alpha = 104{,}55°$

(iii) $A = \frac{1}{2} |\vec{a}| \cdot |\vec{b}| \cdot \sin(\alpha)$

$= \frac{1}{2} \sqrt{45} \cdot \sqrt{69} \cdot 0{,}97$

$= 27$ FE

(iv) a) $\vec{a} = 1\vec{e_x} - 2\vec{e_y} \rightarrow \vec{a} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\vec{b} = \begin{pmatrix} x \\ y \end{pmatrix}$

$\vec{b}$ ist orthogonal zum $\vec{a}$ $\rightarrow \vec{b} \perp \vec{a}$

$\vec{b}$ ist Einheitsvektor $\rightarrow |\vec{b}| = 1$

$\Rightarrow \begin{cases} \vec{b} \circ \vec{a} = 0 \\ |\vec{b}| = 1 \end{cases}$

$\Rightarrow \begin{cases} x - 2y = 0 \\ \sqrt{x^2 + y^2} = 1 \end{cases}$ $\Leftrightarrow \begin{cases} x = 2y \\ x^2 + y^2 = 1 \end{cases}$

$\Leftrightarrow \begin{cases} x = 2y \\ (2y)^2 + y^2 = 1 \end{cases}$ $\Leftrightarrow \begin{cases} x = 2y \\ y^2 = \frac{1}{5} \end{cases}$

$$(\Rightarrow \begin{cases} y = \frac{1}{\sqrt{5}} ; x = \frac{2}{\sqrt{5}} \\ y = \frac{-1}{\sqrt{5}} ; x = \frac{-2}{\sqrt{5}} \end{cases}$$

$$\Rightarrow \vec{b_1}\begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix} ; \vec{b_2}\begin{pmatrix} \frac{-2}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} \end{pmatrix}$$

b) $\vec{c}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

$$\vec{a} = 2\vec{e_x} - 5\vec{e_y} + 3\vec{e_z} \rightarrow \vec{a} = \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$$

$$\vec{b} = 1\vec{e_x} + 1\vec{e_y} - 2\vec{e_z} \rightarrow \vec{b} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$\begin{cases} \vec{c} \perp \vec{a} \\ \vec{c} \perp \vec{b} \\ |\vec{c}| = 1 \end{cases} (\Rightarrow \begin{cases} \vec{c} \cdot \vec{a} = 0 \\ \vec{c} \cdot \vec{b} = 0 \\ \sqrt{x^2+y^2+z^2} = 1 \end{cases}$$

$$(\Rightarrow \begin{cases} 2x - 5y + 3z = 0 \quad (I) \\ x + y - 2z = 0 \quad (II) \\ x^2 + y^2 + z^2 = 1 \quad (III) \end{cases}$$

$$(\Rightarrow \begin{cases} 2x - 5y + 3z = 0 \\ 2x + 2y - 4z = 0 \quad (II) \cdot 2 \\ x^2 + y^2 + z^2 = 1 \end{cases}$$

$$(\Rightarrow \begin{cases} x + y - 2z = 0 \\ -7y + 7z = 0 \\ x^2 + y^2 + z^2 = 1 \end{cases}$$

$$(\Rightarrow \begin{cases} x + y - 2z = 0 \\ y = z \\ x^2 + y^2 + z^2 = 1 \end{cases}$$

$$(\Rightarrow \begin{cases} y = z = x \\ 3x^2 = 3y^2 = 3z^2 = 1 \end{cases}$$

$$(\Rightarrow \begin{cases} x = y = z = \frac{1}{\sqrt{3}} \\ x = y = z = \frac{-1}{\sqrt{3}} \end{cases} \Rightarrow \vec{c_1}\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} ; \vec{c_2}\begin{pmatrix} \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} \end{pmatrix}$$