

+ Normalenvektor: (senkrechter Vektor <sup>on  $\vec{a}$</sup>  zum  $\vec{b}$ )

$$\vec{b}_n = \vec{b} - \vec{b}_a = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} -9/26 \\ 21/26 \\ -18/13 \end{pmatrix} = \begin{pmatrix} -17/26 \\ 51/26 \\ -34/13 \end{pmatrix}$$

$$\Rightarrow \vec{a} = \begin{pmatrix} 3/2 \\ 3 \\ -3/2 \end{pmatrix} + \begin{pmatrix} -17/26 \\ 51/26 \\ -34/13 \end{pmatrix} = \vec{b}_a + \vec{b}_n$$

Aufgabe 14:

$$(i) \quad \vec{a} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \quad \vec{s} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\|\vec{s}\| = 1 \quad (\Leftrightarrow) \sqrt{a^2 + b^2 + c^2} = 1 \quad (\Leftrightarrow) a^2 + b^2 + c^2 = 1$$

$$\vec{s} \perp \vec{a} \quad (\Leftrightarrow) \vec{s} \cdot \vec{a} = 0$$

$$\Leftrightarrow a + b + 2c = 0$$



$$\vec{s} \perp \vec{b} \Leftrightarrow \vec{s} \cdot \vec{b} = 0$$

$$\Leftrightarrow 2a + b + c = 0$$

$$\begin{cases} a + b + 2c = 0 & (I) \\ 2a + b + c = 0 & (II) \\ a^2 + b^2 + c^2 = 1 & (III) \end{cases}$$

$$(I) - (II) \Rightarrow -a + c = 0 \Leftrightarrow c = a$$

$$2(II) - (I) \Rightarrow 2a + b = 0$$

$$(I) - \frac{1}{2}(II) \Rightarrow \frac{1}{2}b + \frac{3}{2}c = 0 \Leftrightarrow b = -3c$$

$$(III) \Rightarrow c^2 + (-3c)^2 + c^2 = 1$$

$$\Rightarrow c^2 + 9c^2 + c^2 = 1$$

$$\Rightarrow 11c^2 = 1 \Rightarrow c = \pm \frac{1}{\sqrt{11}} = a$$

$$\Rightarrow b = \mp \frac{3}{\sqrt{11}}$$

$$\begin{cases} \vec{s}_1 = \begin{pmatrix} \frac{1}{\sqrt{11}} \\ \frac{3}{\sqrt{11}} \\ -\frac{1}{\sqrt{11}} \end{pmatrix} \\ \vec{s}_2 = \begin{pmatrix} \frac{1}{\sqrt{11}} \\ -\frac{3}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \end{pmatrix} \end{cases}$$

$$(ii) \vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0$$

$$\begin{aligned} \|\vec{a}\| &= 1 \Rightarrow \sqrt{x^2 + y^2 + z^2} = 1 \Leftrightarrow x^2 + y^2 + z^2 = 1 \\ \vec{a} \cdot \vec{c} &= \|\vec{a}\| \cdot \|\vec{c}\| \cos(\vec{a}, \vec{c}) \end{aligned}$$

$$\Rightarrow 6x + 3y + 2z = \sqrt{x^2 + y^2 + z^2} \cdot \sqrt{6^2 + 3^2 + 2^2} \cdot \cos 45^\circ$$

$$\Rightarrow 6x + 3y + 2z = 1 \cdot 7 \cdot \frac{\sqrt{2}}{2} = \frac{7\sqrt{2}}{2}$$

$$\begin{cases} 6x + 3y + 2z = \frac{7\sqrt{2}}{2} \\ x^2 + y^2 + z^2 = 1 \\ 2x + 2y + z = 0 \end{cases}$$

$$\left( \begin{array}{ccc|c} 2 & 2 & 1 & \frac{7\sqrt{2}}{2} \\ 6 & 3 & 2 & 0 \end{array} \right) \xrightarrow{2z - 3y} \left( \begin{array}{ccc|c} 2 & 2 & 1 & \frac{7\sqrt{2}}{2} \\ 0 & -3 & -1 & -\frac{21\sqrt{2}}{2} \end{array} \right)$$

$$-3y - z = \frac{-21\sqrt{2}}{2} \Rightarrow z = -3y + \frac{21\sqrt{2}}{2}$$



$$2x + 2y + z = \frac{7\sqrt{2}}{2}$$

$$\Rightarrow 2x + 2y - 3y + \frac{7\sqrt{2}}{2} = \frac{7\sqrt{2}}{2}$$

$$\Rightarrow x = \frac{1}{2}y - \frac{7\sqrt{2}}{2}$$

$$x^2 + y^2 + z^2 = 1 \Leftrightarrow \frac{1}{4}(y - 7\sqrt{2})^2 + y^2 + \frac{1}{4}(7\sqrt{2} - 6y)^2 = 1$$

$$\Rightarrow \frac{1}{4}y^2 - \frac{7\sqrt{2}}{2}y + \frac{49}{2} + y^2 + \frac{49}{4}9y^2 - 63\sqrt{2}y + \frac{441}{2} = 1$$

$$\Rightarrow \frac{41}{4}y^2 - \frac{133\sqrt{2}}{2}y + 244 = 0$$

$$\Delta = \left(-\frac{133\sqrt{2}}{2}\right)^2 - 244 \cdot 4 \cdot \frac{41}{4}$$

$$= -\frac{2319}{2} < 0$$

$\Rightarrow$  keine Lösung

$\Rightarrow \vec{a}$

(iii) A(2|1|3)

B(1|1|2)

C(-1|-1|4)

D(2|-2|1)

$\vec{AB} = (-1; 0; -1)$

$\vec{AC} = (-3; -2; 1)$

B, C, A  $\in E \Rightarrow E: \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + s \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix} \quad s, t \in \mathbb{R}$

$\vec{n}_E = \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ -1 & 0 & -1 \\ -3 & -2 & 1 \end{vmatrix} = \begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix}$

$0 = \begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x-2 \\ y-1 \\ z-3 \end{pmatrix} = -2(x-2) + 4(y-1) + 2(z-3)$

$= -2x + 4 + 4y - 4 + 2z - 6$

$= -2x + 4y + 2z - 6$



$$D \in E \Rightarrow -2 \cdot 2 - 2 \cdot 4 + 2t - 6 = 0$$

$$\Rightarrow -4 - 8 - 6 + 2t = 0$$

$$\Rightarrow 2t = 18$$

$$\Rightarrow t = 9$$

Aufgabe 15 :

$A(4|3|2)$  ;  $B(2|3|4)$  ;  $C(1|1|1)$

(i)

<del>1</del>	<del>4</del>	<del>2</del>	<del>1</del>
<del>2</del>	<del>3</del>	<del>3</del>	<del>1</del>